



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION:	Bachelor of science in Applied Mathematics and Statistics		
QUALIFICATION CODE:	07BAMS	LEVEL:	7
COURSE CODE:	NUM702S	COURSE NAME:	NUMERICAL METHODS 2
SESSION:	JANUARY 2019	PAPER:	THEORY
DURATION:	3 HOURS	MARKS:	90

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS
<ol style="list-style-type: none">1. Answer ALL the questions in the booklet provided.2. Show clearly all the steps used in the calculations. All numerical results must be given using 4 to 5 decimals where necessary unless specified otherwise.3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1 [25 Marks]

1-1. Establish the Padé approximation $e^x \approx R_{3,3}(x) = \frac{120 + 60x + 12x^2 + x^3}{120 - 60x + 12x^2 - x^3}$ [13]

1-2-1. Find the Fourier sine series for the 2π -periodic function $f(x) = x(\pi - x)$ on $(0, \pi)$. [9]

1-2-2. Use its Fourier representation to find the value of the infinite series. [3]

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} + \dots$$

Problem 2 [20 Marks]

For any non negative interger n the Chebyshev polynomial of the first kind of degree n is defined as

$$T_n(x) = \cos [n \cos^{-1}(x)], \text{ for } x \in [-1, 1].$$

2-1. Show the following property [10]

$$\langle T_m, T_n \rangle = \int_{-1}^1 \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0, & m \neq n, \\ \pi, & m = n = 0, \\ \frac{\pi}{2}, & m = n \neq 0. \end{cases}$$

2-2. The property in 2-1. allows us to define the Chebyshev series of $f(x)$ as follows

$$f(x) \sim \sum_{k=0}^{\infty} c_k T_k(x) = \frac{1}{2}c_0 T_0(x) + c_1 T_1(x) + c_2 T_2(x) + \dots$$

where $c_k = \langle f, T_k \rangle / \langle T_k, T_k \rangle$ for $k \geq 1$ and $c_0/2 = \langle f, T_0 \rangle / \pi$

2-2-1. Determine the Chebyshev series expansion of $f(x) = \sqrt{1-x^2}$. [10]

Problem 3 [19 Marks]

3-1. Given the integral

$$\int_{0.04}^1 \frac{1}{\sqrt{x}} dx = 1.6$$

3-1-1. Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2, 3, 4$ using the recursive Trapezoidal rule. [15]

3-1-2. State the two-point and the three-point Gaussian quadrature rules respectively for a continuous function f over the interval $[-1, 1]$. [4]

Problem 4 [26 Marks]

4-1. Assume a 3×3 matrix A is known to have three different real eigenvalues λ_1, λ_2 and λ_3 . Assume we know that λ_1 is near -2 , λ_2 is near -5 and λ_3 is near -1 .

4-1-1. Explain how the power method can be used to find the values of λ_1, λ_2 and λ_3 respectively. [2×3=6]

4-1-2. Discuss how shifting can be used in 4-1-1. to accelerate the convergence of the power method. [2]

4-2. Use Jacobi's method to find the eigenpairs of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

[18]

God bless you !!!